A distance in-service teacher education setting focused on mathematics investigations: The role of reflection and collaboration[1]

João Pedro da Ponte
Leonor Santos
Departamento de Educação da Faculdade de Ciências da Universidade de Lisboa

Resumo. Este artigo discute como diversos professores participaram num curso de formação a distância inspirado na noção de investigação pelos professores. Os participantes são professores dos 2.º e 3.º ciclos do ensino básico e do ensino secundário que se inscreveram e trabalharam com um parceiro. Apresentamos estudos de caso de três grupos de professores, anotando o seu desenvolvimento profissional no que respeita a investigações matemáticas, reflexão and práticas de colaboração. Os três casos mostram experiências notavelmente diferentes. Alguns professores ajustaram-se bem ao formato e actividades propostas mas outras tiveram dificuldade em assumir um papel activo. Concluímos que ler, discutir, realizar tarefas de natureza aberta, reflectir e colaborar são actividades poderosas mas requerem a uma certa preparação prévia.

Abstract. This paper discusses how several teachers were involved in a distance in-service course inspired by the notion of teachers' inquiry. The participants are middle and secondary school teachers who register and work with a partner. We provide case studies of three groups of teachers, tracing their professional development concerning mathematical investigations, reflection and collaboration practices. The three cases show strikingly different experiences. Some teachers adjusted well to the format and activities proposed but others had trouble in assuming an active role. We conclude that reading, discussing, doing open-ended tasks, reflecting and collaborating are powerful activities but require a specific readiness.

Palavras-chave. Formação de professores de Matemática, Formação a distância, Formação contínua de professores, Colaboração, Reflexão, Inovação curricular.

Kewwords. Mathematics teacher education, Distance teacher education, Inservice teacher education, Collaboration, Reflection, Curriculum innovation.

1. Introduction

In Portugal, a new mathematics curriculum for basic and secondary education was established in 1991, emphasizing problem solving, applications of mathematics, and mathematical reasoning. The secondary school curriculum was again revised in 1997, resulting in a stronger emphasis on the use of technology (especially graphic calculators) and suggesting that pupils should carry out mathematical investigations – for example, in the study of functions. Mathematical investigations may be regarded as closely related to open-ended problem solving, modelling activities, and mathematical projects. Whereas exercises and problems tend to be highly structured, framing the activity of pupils in a predictable way, investigations leave some room for them to ask their own questions and follow different paths (Ponte, 2001). This way pupils' activity

[1] Ponte, J. P., & Santos, L. (2005). A distance in-service teacher education setting focused on mathematics investigations: The role of reflection and collaboration. *Interactive Educational Multimedia, 11*, 104-126. (retirado de http://www.ub.es/multimedia/iem/ em 21.Fev.2006).

becomes more similar to that of the research mathematician. Investigations provide a stimulating context for pupils' thinking, requiring them to justify their reasoning before teacher and colleagues (Mason, 1991) and they are good starting points for inquiry-based mathematical classes (Wood, 1994). As pupils confront their different conjectures and justifications, they work as a small mathematical community, where mathematical knowledge develops as a common undertaking.

However, using investigations in the classroom poses several demands on the teacher's knowledge and competencies. Teachers must have a good personal relation with mathematical investigations, that is, they need to know what a mathematical investigation is, how it is carried out, and how its results are validated and they need to feel confident in carrying them out. To make sense of them, teachers need a general view of mathematics that is not restricted to definitions, procedures and rules, but that values exploratory activity. Also, teachers must know how to use investigations in their professional practice, including (i) knowing how to select and adapt exploratory and investigative tasks adjusted to their classes; (ii) knowing how to direct pupils carrying out investigative work in the classroom, in the phases of introduction, development of the work and final discussion; (iii) having confidence in their capacity to manage the class atmosphere and the relations with pupils to carry out this work; and (iv) carrying out curriculum planning and management that includes explorations and investigations in a balanced way, regarding the characteristics of their classes (Ponte, in press). Taken together, these are demanding issues. Teachers need to develop their competencies in this respect and specific teacher education initiatives are needed to support them.

This paper is based on a distance in-service teacher education course designed to address the classroom use of mathematical investigations. This teacher education activity is itself inspired by the notion of teachers' exploration and inquiry, uses a flexible pedagogy and is based on the notions of collaboration and reflection. We present and discuss the case studies of three groups of teachers, discussing their journeys and achievements in this virtual setting and tracing their professional development concerning mathematical investigations, reflection and collaboration practices.

2. Reflection and collaboration in distance teacher education

For mathematics teachers, especially those living in remote areas, distance education is a useful framework to provide in-service teacher education opportunities.

However, distance education can be designed according to a wide variety of pedagogical perspectives. The most common approach is to follow a highly structured format, specifying objectives and sub-objectives in detail; tasks are then designed to fit these objectives, assessing each one in turn and moving forward only when a subset of objectives is met. Another approach is to design distance education as a framework for flexible learning (Collis & Moonen, 2001), regarding teachers as the main agents of their professional development, supported by an environment rich in challenges and interactions.

This course adopted the second perspective, viewing professional knowledge as including knowledge of mathematical content, curriculum, pupils' learning and instruction. We regard professional knowledge as integrated, oriented towards action (Elbaz, 1983), and constructed by reflection on and about practice (Ackerman, 1993; Shulman, 1987). In such a perspective, in-service teacher education is most effective when contextualized in teachers' professional activity (Hargreaves, 1994; Smylie, 1995). Teachers' professional development may draw on professional collaborations involving projects, explorations, and reflections and on participation in the professional culture by attending meetings, talking with other teachers, and reading the professional literature and formal and informal teacher education opportunities may be combined to support it.

The distance education course presented in this paper assumes that the context for professional development regarding the use of mathematical investigations may draw on interaction among several partners such as teachers, teacher educators, and machines with teachers involved in inquiry-oriented activities like working on projects and exploring links, bibliographic resources, software, and online documents. Another important feature of this virtual course is its stress on reflection and collaboration.

A reflective teacher may be regarded as a teacher who questions his or her activity, who wants to know if pupils learned or not what was intended, what went right or wrong in the class, the reasons that may lie behind the success or failure of pupils, and the ways to overcome problems. However, reflection may be oriented to change or an intention to keep the *status quo*. That is, reflection may be used to question action as well as to justify it, defending the teacher from criticisms. Zeichner (1993) indicates different traditions in which reflection plays an important role. In the academic tradition, reflective practice concerns the translation of the knowledge of disciplines for the development of teachers' understanding; in the tradition of social efficiency, reflection is used to apply teaching strategies derived from educational research; the developmen-

tal tradition stresses reflection regarding pupils' cognitive development; and, finally, the tradition of social reconstruction stresses reflection regarding the social and political context of the school. This wide spectrum of meanings shows that the nature and the quality of reflection are more important than its simple occurrence.

The work of John Dewey constitutes the main reference to most discussions on teacher reflection. In his book *How we think*, he distinguishes between routine act, led by impulse, habit or submission to authority and reflexive or questioning act, based on will and intuition, implying the search for logical and rational solutions to problems. He states that reflective thinking is the "active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusion to which it tends" (Dewey, 1997, p. 6).

For this author reflection is based on the scientific method of analysis of problems. Mewborn (1999) describes in the following way the different phases that Dewey considers in the reflective process:

- Recognizing that a given situation is problematic and may have several solutions;
- Problematizing the situation, identifying the conditions that bear on the problem;
- Generating solution hypotheses, collecting data to refine some and eliminate others;
- Reasoning about hypotheses;
- Testing hypotheses.

Donald Schön (1983) is another author who discusses the notion of reflection. He considers reflection in action, reflection on action, and reflection about reflection on action. This last one is fundamental for the development of teachers' professional knowledge, as it allows a deeper analysis of problems and promotes the emergence of new concepts and perspectives. This author critiques the shortcomings of 'technical rationality', regarded as the attempt to apply the results of social science to professional practice. He proposes an 'epistemology of practice' as a new kind of rationality taking professional practice as the starting point. In his perspective, the issue is not that of applying science to practice but interrogating practice as a rather specific style that he calls *'artistry'*. This draws on the contributions of science, when possible, but it is much more complex and personal than just the application of science results.

Oliveira and Serrazina (2002) distinguish among three concepts: (i) reflection,

(ii) reflective teacher, and (iii) reflective practice. They suggest that the key issue is to look at the teachers' practice and to ask if it is reflexive or not. In their view, the emphasis must be neither on whether the teacher as a person deserves the label of "reflexive" nor whether the thinking processes are really "reflection" – instead it must put on analyzing teachers' practices.

Mewborn (1999) underlines that reflexive practice is not contemplative but clearly oriented towards action. She indicates that the following aspects are relatively consensual regarding the way most authors regard reflection:

> - Reflection is qualitatively different from recollection or rationalization […];
> - Action is an integral part of the reflexive process. Neither *verbalism*, which is reflection without action, nor *activism*, which is action without reflection, is sufficient to constitute reflective thinking […];
> - Reflection is both an individual and shared experience. (p. 317, original emphasis)

As reflection always focuses on some problem, we need to consider possible objects and modes of reflection. As objects of attention, teachers may take some specific aspects of their practice or more general issues. More importantly, they may think about classroom problems, trying to fix them, or question their core perspectives and reframe the issues in a different light – what several authors refer to as critical reflection (Jaworski, 1993; Serrazina, 1998). The starting point for a critical reflection may be a problem concerning learning fractions by a particular pupil as well as the reasons that lead a certain group of pupils with a given cultural background to not participate in classroom activities. The important point is how that reflection is carried out and the implications that are drawn. Studying that requires understanding teachers' priorities and modes of reflection, considering the nature of the issues brought to attention, the questioning depth of the reflection, and the commitment in searching for solutions.

Collaboration is another theme that is attracting much interest in the mathematics education literature (see Peter-Koop, Santos-Wagner, Breen, & Begg, 2003). It concerns the activity carried out by several actors, in homogeneous groups (for example, teachers from some school level) or heterogeneous groups (for example, mathematics teachers and university teacher educators). However, a high degree of affinity and mutuality among all participants is required so that an activity may be regarded as collaboration; otherwise, it will be an instance of joint work or cooperation rather than collabo-

ration (Wagner, 1997).

In a collaborative activity all participants contribute to the achievement of common goals, according to their capacities and expertise. This is a way of work particularly suited to deal with complex problems, that a single person would be unable to solve; however, within a group it may be possible to deal with such problems and this process provides each participant with the opportunity to grow professionally. Besides the common goals, the different participants may have their own personal objectives, but they try to adjust them to the interests of the group. In collaboration, dialogue is the essential feature that frames the relationships among participants (Erickson, 1989). Those who know something teach those who do not know and they all learn new things as they work together, regarding the specific tasks, regarding others, and regarding themselves: "As we learn more from and about others, we also learn more from and about ourselves" (Olson, 1997, p. 25).

Collaboration is very powerful but it is also a complex process to initiate and sustain. Frequent meetings of teachers may be not sufficient to implement new practices (Ellis, 1990). As Little (1990) indicates, instead of being a factor of change, working together may contribute towards complacency, leading to the reinforcement of existing practices. This author presents four kinds of collegial relations among teachers and arranges them in a continuum that progressively represents stronger ways of collaboration. The weakest is scanning. The second is storytelling, followed by support and assistance. Finally, she considers joint work as the most promising form of work among teachers for changes in practice. In this way, certain forms of collaboration may also contribute towards a conformist attitude, leading to the position dominated by the group, erasing individuality, promoting co-optativism, and assuring teachers' acceptance of educational reforms with which they do not agree with (Hargreaves, 1994). As cooperation is based on a relational context, attention needs to be paid to the possible uneven distribution of power.

To develop a productive collaboration is a promising but demanding process:

> Collaboration is not easy. The tensions which emerge in collaborative relationships are what keep the relationships alive and dynamic. There are no easy or sure answers. In collaboration, questions become the focus rather than only answers. When it is not assumed there is one correct answer or one right way, questions lead to understanding as we each become researchers of our own personal/professional knowledge and practice. (Olson, 1997, p. 25)

Collaboration is a valuable resource for professional development; but in order to provide an opportunity for learning and change, a collaborative activity must be a dynamic process, involving negotiation, voluntary commitment, and valuing of differences. Based on a relationship of at least relative equality, in which each person has value, collaboration is an empowering source of professional learning, enabling us to increase our self-confidence, reduce uncertainty, encourage risk-taking, and to support failure and frustration (Hargreaves, 1994). Collaboration does not exclude individual work. Therefore, there is not a necessary opposition between these two forms of work; rather, they may complement and reinforce each other (Santos, 2002).

3. The distance teacher education course

This in-service distance teacher education course is carried out at the University of Lisbon under the name "Learning mathematics by investigating" (see http://ia.fc.ul.pt/ce). The aim is to offer teachers some theoretical ideas and practical experience in working with mathematical investigations in the classroom and to contribute towards their professional development, providing opportunities for reflecting on their own practice, for developing a culture of collaboration, and to become better acquainted with information technology. This in-service activity lasts for six months and works as a "study group" – each group has twelve teachers and one or two teacher educators. It is divided in three segments: (i) dynamics of the mathematics classroom; (ii) investigations in mathematics and in professional practice; and (iii) experimenting with investigations in the classroom.

The participants are middle, junior high, and secondary school teachers[2]. Registration was not individual, but in pairs; that is, teachers had to enroll in the course with a partner of their own choice. These pairs were the basic working unit throughout the activity. The participants come from different regions in Portugal (2 from the rural North, 8 from Porto; 6 from the Centre; 13 from the Lisbon area, 4 from the city of Lisbon; 1 from Alentejo) and 2 are from Brazil. Of the 36 who began, 2 dropped out and 34 completed the program successfully.

[2] In Portugal, basic education is compulsive and includes three cycles: First cycle or primary school (grades 1-4), Second cycle or middle school (grades 5-6), and Third cycle or junior high school (7-9). Secondary education or high school (grades 10-12) is not compulsive.

The setting designed for this course includes a Web environment through which various materials are provided. For each segment, there is a study guide and several papers – some constitute required reading and others are optional. These papers were to be read and discussed by each teacher with their partner, with other participants in a discussion forum, and if necessary with the teacher educator. Some papers were written for this course and others were drawn from the professional and academic literature and all of them were presented in Portuguese (original versions or translations). Examples of required papers are Fonseca, Brunheira & Ponte (1999), Poincaré (1996), Ponte, Boavida, Graça and Abrantes (1997) and Skovsmose (2000).

There were also five tasks and a final questionnaire that the pairs of teachers had to undertake and send to the teacher educator, who provided feedback. The tasks were open and diversified: in task 1, the teachers had to comment on one of the required papers; in task 2, they had to describe and analyze a classroom situation that they had experienced; in task 3, they had to select and analyze a Web site relevant to mathematics investigations; in task 4, they had to study a problem from the history of mathematics; and, in task 5, they had to design a mathematical investigation, use it in their classroom, and reflect on this experience. The tasks were marked as "Good", "Sufficient" or "Unacceptable" and in the latter case a reformulation would be requested.

This in-service course has three sections (two on the topics of numbers and functions and one on geometry), organized according to the teachers' preferences. The first session (3 hours) and the last session (6 hours) are face-to-face meetings held at the University of Lisbon. Different kinds of interactions may be established among the participants:

- Teachers interact with their partner teacher, as they work collaboratively;
- Teachers interact with the system, downloading materials and looking for information on the Web site and elsewhere;
- Teacher educators and teachers interact face to face (in the first and last sessions);
- Teachers interact with teacher educators, via email and the Web site, sending tasks, answering questions, and reporting their progress;
- Teachers and teacher educators interact in a discussion forum.

Besides the participating teachers and teacher educators, the course involves a coordinator, a technician who takes care of the Web environment, and a team of two external evaluators.

For each teacher, the final evaluation involves three main aspects: carrying out the tasks, participating in the discussion forum, and self-evaluation. This self-evaluation was done through a final questionnaire with open response questions on the activity of the course and a self-assessment.

Research methodology

The distance education course was subject to both internal and external evaluation. The internal evaluation was carried out in an ongoing way by a coordinating team that included the coordinator and the teacher educators. They were permanently in contact by telephone and in a closed discussion forum and had regular meetings to reflect about the development of the activity.

The external evaluation involved two parts. One was a general assessment of the whole course, which covered all the participants. The other concerned the case studies of three groups of teachers and focused on the virtual setting – especially the interactions, the tasks and the papers provided – and on the effects of this activity on the participants' professional development. The groups of teachers for the case studies were selected according to the mathematical topics they chose (Geometry or Numbers and Functions), and to the years of professional experience of its members.

For the general assessment, a detailed analysis of teachers' final questionnaires was carried out, as well as of other documents and notes of participant observation of teacher educators' activities. In the case studies, data was collected through semi-structured interviews with each pair of teachers; an interview was carried out with the course still in progress and the other just after it finished. Data was also collected from the teachers' papers, final questionnaire, and messages exchanged with the teacher educators. In this paper we draw especially on the three case studies, taking also into consideration the general results of both the internal and external evaluations. In our analysis we begin with a brief presentation of each pair of teachers, describe their journey throughout the course in relation to mathematical investigations, and address the role of reflection and collaboration in this professional development experience.

Teachers' journeys in the course

Isaura and Anabela

Isaura has a degree in mathematics teaching. She has been teaching for fifteen years in basic and secondary education (in recent years just in secondary education). She often attends the mathematics teachers' annual meeting (ProfMat). Anabela is a young teacher who completed the same degree just two years ago; during the practicum, Isaura was her mentor. When she took this course, Anabela was teaching the 3rd cycle of basic education. These teachers work in different schools in the South of Lisbon area.

Activity in the course. In task 1, Isaura and Anabela decide to comment a text about learning activities (APM, 1988). However, the teacher educators consider that their comments do not address what was proposed in task and ask for a reformulation. The teachers accept this and add two more paragraphs. In task 2 the teachers describe a 12th grade class about inverse functions, detailing an episode where they make a link between the daily life notion of inverse and its mathematical meaning. In task 3 they present a site with many links for other sites that the teacher educators assess as of little help concerning investigations. Task 4 asked teachers to present a problem with historical relevance, however, Isaura and Anabela decide to select a question that they justify on other criteria: "We chose the problem of the division of camels because we found it fun and possible to work from primary school on" (Isaura, *interview*). Finally, in task 5, the teachers are advised by the teacher educators to change their initial proposal (construction and use of an astrolabe), as they do not regard this as a mathematical investigation. The teachers settle on a new proposal (exploration of Pick's theorem) and, in their final report, they stress the difficulties felt by pupils and suggest that these should have more orientation: "If we apply this task again in a 7th grade class, similar to this one, it should be more directed, in the sense that pupils would progressively increase the number of nails in the border of the polygon" (*task 5*).

The quality of the work of Isaura and Anabela in the first four tasks, as judged by the teacher educators, was problematic. The teacher educators asked the teachers to reformulate one task and accepted with reservations two others as "sufficient". The teachers seem to take in a very light way what is asked for in the tasks or perhaps they have trouble with the key concepts that are addressed in this course. The problems in carrying out the tasks and with the evaluation received from the teacher educators led the teachers to consider dropping out of the course: "As we got feedback from the other

side [tasks 3 and 4] we wondered: 'is this worthwhile? Here we are working hard one afternoon a week...'" (Isaura, *interview*). However, they regarded the next task as being quite practical, and so decided to continue.

Up to the end of the course most messages exchanged between the teachers and the teacher educators had an administrative nature or concerned the evaluation of the tasks. A more interesting interaction arises only when Isaura and Anabela begin the fifth task and they ask the teacher educators their opinion about the activity to carry out with pupils.

Mathematical investigations. Isaura indicates that as a pre-service teacher she heard about investigations, but did not have a clear idea: "I heard one professor or another who talked about that at the university. Not much. I was a bit confused because I didn't know what it was" (*interview*). As time went by, with her participation in the ProfMat, her ideas began to clarify. Despite that, these two teachers seem to have little former experience concerning classroom use of mathematical investigations. As they explained, one of the reasons they enrolled in this course was their interest in doing new things, particularly in Geometry, where they feel that their practices needed to change: "We end up always doing the same thing, direct problems. And there is little else. Either we are not very creative or we know few things that we can apply to pupils regarding geometry" (Isaura, *interview*). Their main difficulty in designing investigations is related to the open nature of these tasks:

> Isaura: When I am designing an activity, to understand if it is or not an investigation. At the beginning I think it is, but afterwards it is not, because it is too directed.
>
> Anabela: Because it is not open.

Another problem that they refer concerns classroom management. They say that they feel some distress because it is difficult to answer pupils in an immediate way if they follow an unforeseen strategy. However, they believe that with practice they can solve this:

> Isaura: When a teacher gives pupils a worksheet she knows every way of doing it, but when she gives an investigation task she does not, she does not know what ways pupils take (…) and we feel some distress because we do not provide the answer right away. It is this kind of uneasiness that we wanted to be solved in this study circle.
>
> Anabela: We feel that we can handle it with more practice.

The papers written by these two teachers during the study circle do not include deep reflections about mathematical investigations. The reflection they write in task 5 stresses pupils' difficulties and does not suggest any appreciation of the power of this activity. At the end of this study circle, the perspective of these teachers regarding what they learned is positive, although with some reservations. Anabela says that "in a way" she feels more comfortable about using investigation activities in her classes in the future. Isaura considers that this course "clarified some ideas about tasks and mathematical investigation" (*task 6).* The fact that, in task 5, they had to choose a second task to work in the classroom and that they propose to increase the structure of the questions to reduce pupils' difficulties, suggests that the problems they initially felt were not completely overcome.

Reflection. Isaura and Anabela consider that the proposed tasks, especially in the beginning, had mostly a theoretical character and were not directly linked to the mathematical theme. They would prefer that the tasks had a more practical nature:

> If the course were more practical, perhaps that would have been better [I mean], if it had more to do with geometry. Because I found that in the beginning, mostly, it was too theoretical. It was the analysis of papers, comments of papers. In that regard it was a mistake. (Isaura, *interview*)
>
> I think it has to do [with] us wanting to do everything in much more practical terms and they [the teacher educators] in more theoretical terms. (Anabela, *interview*)

These teachers seem to establish a strong dichotomy between theory and practice. They consider that the activities closely related to specific mathematical topics are highly practical. They also think that the first tasks proposed, involving the analysis of papers with classroom situations, belong to the theoretical field. In this manner, they clearly show more interest in reflecting on some issues rather than on others.

Both teachers were sorry that there was not a stronger interaction with other groups of teachers and, especially, with teacher educators. The discussion forum was one of the opportunities for such interaction and reflection provided by this teacher education virtual setting. For these teachers, this forum was used mostly to "discuss texts" (Isaura, *interview*) – which points towards the theoretical field. They come to that conclusion because the teacher educators proposed a question in the forum about one of the

papers – the meaning of the concepts of task and activity – and when the discussion ended they repeated the process:

> I think that what happened was the teacher educators saw that we were not getting started and formulated one question. The first was about task and activity that the groups ended up discussing. When the discussion was over, it was really over. And then they come back to raise another question. What would happen if the teacher educators did not put the questions themselves? I do not know. Would the discussion forum have worked with other themes? (Anabela, *interview*)

The participation of Isaura and Anabela in the discussion forum was rather low. This is no surprise if we note the little interest that they assigned to theoretical issues and the fact that they considered that to be the main purpose of the forum.

Reflection, in this teacher education course, was to be done in great part through writing. However, these two teachers do not seem to be very comfortable about using this form of expression. That is apparent in the tone they use in their messages: "Joint we send…" plus their names. That is in sharp contrast with the open and amicable relationship they show in face-to-face meetings such as the interviews. As they said at some point: "We have trouble expressing our opinion in front of a computer" (Isaura, *interview*).

It seems safe to conclude that these teachers have a problematic relationship with the papers provided in the course and have some difficulty in writing and communicating through the Internet. Besides, they establish a sharp distinction between what they regard as theory and practice and just show interest in reflecting on issues related to mathematics teaching topics.

Collaboration. These two teachers, now working in different schools, had a past history of working together. This relationship left positive marks in both, as Anabela decided to invite her former mentor to join her in this course and Isaura accepted immediately. They met at Isaura's home as Anabela had recently moved and she had no telephone yet.

Isaura and Anabela consider the fact that this course is based on teams of two elements as being quite positive. Without that, in their opinion, working at a distance could become painful: "You're completely isolated" (Isaura, *interview*). They often read the assigned texts individually. At their weekly meetings they discussed these texts and made the proposed tasks:

> We had to read them before and afterwards we discussed them. In the three or four hours peer week that we meet, we were not able to discuss the texts and solve the tasks. We would either do one thing or the other. (Isaura, *interview*)

There were no signs of clear division of labor between them. In the past they had quite asymmetrical roles (mentor/student teacher), but they seem to have a balanced relationship right now. None of these teachers mentioned any difficulty in the teamwork dynamic they developed in these months.

Alda and António

These two teachers had a similiar professional experience. Both have a degree in mathematics teaching, that they did at the same time, and they did the practicum in the same school. They have worked a lot together. In the two years after the practicum they kept collaborating, leading sessions in national meetings of mathematics teachers and writing papers. During the practicum they undertook a joint project about the evaluation of investigation tasks that included the elaboration of a Web page. During this course both of them were teaching in the 3rd cycle of basic education in different schools in the South suburbs of Lisbon.

Activity in the course. In task 1, Alda and António comment on the role of the teacher in creating significant mathematical situations for their pupils, underline the importance of previous reflection, the nature of communication and negotiation to develop in the classroom. They add that teachers need to reflect on their classes, and they do that presenting a self-critique of classes given in previous years. In task 2 they describe an episode from a class in which pupils discuss the conjectures formulated the day before. In their conclusion they reflect on the role of the teacher regarding pupils verifying their conjectures, in searching for a communication adjusted to their characteristics, and in the difficulty in foreseeing pupils' questions. In task 3, they present a Web site stressing the distinction between activity and task. The mathematical problem discussed in task 4 concerns the notion of infinity and Cantor's continuum hypothesis. In task 5, they propose different situations in their classes (they were teaching different school grades) and present two different papers. Alda reports a class in which pupils work in pairs, exploring families of linear functions. She presents a classroom episode where pupils had trouble with some concepts and ends with a reflection about her role

in this class, especially in leading the discussion and managing pupils' work. António proposes to his pupils the solution of an equation in which the first member is a complex power (the basis and the exponent are both 2^{nd} degree polynomials) and the second member is one. He reflects about what he did during the presentation of pupils' work, presenting some self-critiques.

Alda and António carried out the proposed tasks according to the expectations of the teacher educator – a person that they knew quite well as pre-service teacher and university supervisor. In all tasks they receive the mention "Good" and positive comments. In tasks 1 and 3, before beginning their work, they asked for clarifications. They feel that their involvement had a steady decrease along the course, perhaps because they started with very high expectations about the level of the discussions that they did not feel happening. However, they were always quite careful in reflecting about their teaching practice, in particular in the work related to mathematical investigations.

Mathematical investigations. These two teachers were quite aware of many issues regarding mathematics investigations, a notion that they had worked with in their pre-service education, but they felt some difficulty in its implementation in the classroom:

> We regularly produced this kind of task and took them to our classes. However, sometimes we felt that despite our intention being to awaken in pupils a significant mathematical activity this didn't happen, we witnessed [...] the quick solution of the task or giving up, sometimes with sentences such as "I don't understand!" or "this is too difficult!" We had some trouble in explaining the why of this situation since the task was designed with the intent to draw on our pupils' "higher cognitive processes". (*task 1*)

Alda presents (in *task 5*) a classroom episode where pupils had trouble in distinguishing between direction and sense of a line and she ends with a reflection about her role in this class, especially in leading the discussion and managing pupils' work. In this reflection she shows a deep understanding of the key role of discussions when using this task in the classroom and shows awareness of the issues related to the role of the teacher as a source of authority:

> I didn't feel any trouble in leading this discussion, as the interventions of pupils arose naturally. However, I had a lot of trouble controlling the dis-

> cussion time, since the contributions of pupils were many and I felt that it was important to analyze them. (*task* 5)

For example, regarding the validation of results reached by pupils, Alda says:

> Some pupils called me and put me this question and here I made a mistake. Realizing pupils' enthusiasm in discovering five solutions, I could not restrain myself and I told them that they had found all the solutions. This made pupils end the task without understanding the concept of power and when a power is equal to one. For these pupils, this task was just a few more calculating rules. After I reflected on this, I concluded that I should have asked them how they could find other solutions if they existed, or, if they felt that there were no more solutions, how to prove that. (*task 5*)

Related to this same concern with the kind of teacher intervention, António states that the teacher "needs to know when he must let a group go by itself and let it arrive at its own conclusions" (*task* 5).

For Alda and António participation in this distance education course on mathematical investigations helped them to think about many issues related to such classroom activity, thus constituting a valuable professional learning experience.

Reflection. Alda and António were familiar with investigation activities. However, reading the papers of the course – which they enjoyed a lot – led them to think deeper about what is involved in doing this kind of work in the classroom and to reconsider the meaning of the term "investigation activities" and the role of the teacher. From such reflection they took consequences to their practices, trying to carry out classes in a different way:

> António: When we read the first papers we began having a better (...) notion of what investigative character in a class is and what the role of the teacher is.
>
> Alda: And we immediately analysed our classes – I was doing this that way, it shouldn't be – we began to be pay much more attention.
>
> António: The first segment was an eye-opener to a lot of things (...) [We concluded] that all classes must have that investigative character, that attitude of the teacher in attempting that it is the pupil who discovers by him or herself. And then we started to generalize the concept of investigation and then our classes have been changing gradually. (...) The first segment was very good because of that and the texts were excellent, both the required and the recommended ones (…)

> Alda: We concluded, at least in my case, and we have also already discussed that we didn't do it in the best way (...) The first [segment] is essential so that we begin to question ourselves, to try to change. Afterwards, when we attempt to change, we want to analyse whether our change was positive or not. (*interview*)

These teachers stress the fact that this course enabled them to work in a more autonomous way than the courses they attended before. They recognize that the teacher educator not being around led them to discuss more with each other trying to come up with a conclusion:

> I felt that what I learned it was I who read it, it was I who went exploring. And that is good because the teacher educators provided indications, provided all the bibliography, the sites, that was very important, and then, from there we did the exploration. But, I mean, there was no direct learning from the teacher educators, right? It was not a course in which there is someone giving direct instructions, talking, and so on. So, they provide indications about what we must do and that is good because it creates autonomous work, but on the other hand I felt a bit lost sometimes – am I doing this all right? Should I do better? (António, *interview*)

They refer that they looked to solve the doubts they had during the course among them. For example, regarding a doubt that arose in one of Alda's classes, they formulated intervention hypotheses and put them into practice:

> We tried to solve the questions among us, I think. We tried to find hypotheses and so… I remember one time the doubts of this class I had. We then ended up with more or less some idea how to solve it. (Alda)

They not only showed interest in reflecting about the questions that they face in their practice as they showed interest in exploring the consequences of their reflections in their own classes.

Contrary to Isaura and Anabela, these two teachers assume that they enjoyed reading and writing texts. Apparently, they feel at ease with writing and take pleasure in using it for professional purposes:

> It was always positive because of what we learned, and making us work, write things, write papers, read things. If there were not these stimuli we would have ended up not doing it. We don't have to. That alone makes it good. (António, *interview*)

These teachers, in a message sent to the teacher educators when they finished task 2, decided to present and question a set of problematic situations that they experienced in their classes. This message was written after a short interruption of classes that provided them with the opportunity to read some papers, to reflect more and select questions. Furthermore, in the beginning of the course, they sent some contributions to the discussion forum that may be regarded as interesting examples of reflections. A first message is based on the text that they read and they refer that these papers indicate that teachers must be careful in order not to corrupt the investigative character; they also helped them to understand that the investigative character must be present in every mathematics class. In another message they suggested the reading of texts from a book related to these topics, presenting their main ideas. They still had another four interventions on the discussion forum, responding to messages from other participants.

Collaboration. These two teachers had a previous experience of joint collaborative work. They have a positive view of the work that they did together in this course, recognizing its advantage regarding individual work, notably in the discussion of papers, clarifying doubts, carrying out tasks. They also note the potential of working together to maintain a high motivation.

> António: In our case it was great.
>
> Alda: I think that we would have become unmotivated more quickly.
>
> António: And even to discuss papers.
>
> Alda: Yes, and the thing about those doubts as well [… Doing things] just by myself is more complicated.
>
> António: And the tasks become easier to do.

António thinks that some of the tasks proposed in the course could have been done individually but in that case "there would be not so much discussion and the enrichment of knowledge would be less".

The working dynamic of these two teachers was marked by some autonomy and developed in a natural way, without any serious difficulty. In the interviews, they appeared to maintain a balanced relationship. In two situations they voiced different positions in a natural way. All their work was carried out jointly, except a comment sent to the discussion list by António.

Júlia and Maria

Júlia has 32 years of experience in secondary school mathematics teaching; she has a degree in mathematics and a master in mathematics teaching and she is also an in-service teacher educator. Maria has 22 years of experience and she is on a sabbatical leave to do her master thesis, on the use of the Internet in the classroom; she has a degree in mathematics teaching and she usually teaches in secondary school. Presently both are accompanying teachers for the new secondary school curriculum near the city of Porto, in the North of the country.

Activity in the course. In task 1, the teachers did not select a paper to analyze but decided to do a written comment based on several papers that they read. In this comment they emphasize that teachers need to know how to "ask questions" in the classroom, respecting or not their pupils' logic. They also call attention to the fact that teachers need to know their pupils very well and refer certain curricular, organizational and material constraints on teaching practice:

> It is important to know how to ask questions stimulating the pupil to progress without finding in our intervention a reflection wall towards another direction, that he does not desire [...] It is the management of such experience with curricula, with bells, with or without resources and everything else that sometimes produces cuts, discontinuity in learning [...] Knowing how to ask questions, maybe involves a capacity to put ourselves at the level of knowledge and involvement of the pupil, including the affective side. (*task 1*)

In task 2, Júlia and Maria presented an episode from the classroom of one of them where pupils had to look for conditions to define planes and lines. They also analyzed the behavior of the teacher, valuing the opportunity that she gave a pupil to present his strategy. In task 3 they commented a Web site on "Women and mathematics" that they justified on the grounds that it enables pupils to investigate "inside and outside the class, in a more or less autonomous way, individually or in groups, answering mathematical challenges and involving him or her in a work with transdisciplinar potential" (*task 3*). For task 4 they selected a conjecture of Kepler regarding the minimum volume of a pile of spheres; they presented the history of this problem and its resolution and ended up discussing its use in secondary school mathematics. Finally, in task 5, they used a book of mirrors to study the notion of sequence. The first part included a study

visit to an exhibition, where pupils contacted with fifteen investigation tasks. They needed to pick one and, in pairs, work on it and present their reports. The second phase was in the classroom. The teachers' report finishes with a reflection about their experience in this course, including some implications for their professional practice, indicating the demands of these classes and also their multiple advantages. This experience was later reported in a professional meeting, in a session that, in their words, was "much attended" that they recall with great satisfaction.

These two teachers seem to have understood quite well what was intended in this course and carried out the proposed activities with high enthusiasm and creativity. The teacher educator marked all their papers as "Good" and was very positive in his comments. Júlia and Maria sent several messages to the discussion forum and maintained a fluent exchange of messages with the teacher educator, with whom they developed a friendly relationship.

Mathematical investigations. These teachers were quite aware of the potential of the mathematical investigations that they often used with their pupils. The discussions that they carried out during the course, within the group and in the discussion forum, helped them to reflect deeper on issues related to working with this kind of task in the classroom. The work of this course appeared to bring extra support to continue to carry out such activity with their pupils. The kind of development that they did in task 5 led these teachers to include, among the competencies that they developed, establishing connections between different mathematical topics. As they explain: "Starting with an exposition that in the beginning one could expect to be of (strict) geometrical nature, it was possible to introduce the theme of sequences, review function concepts and much more" (Maria, *task 6*). Júlia also explains that "we learned in a deep way that mathematical connections are the mathematics we should teach" (Júlia, *task 6*).

These teachers were able to integrate the work that they had to develop in this study circle with the work that they had to do in supporting other teachers with the new mathematics curriculum. This is another aspect that they emphasize:

> We were able to integrate all the work in such a way that behind everything that we prepared for the study circle was the intention of being used either in our classes or in the activities that we were developing [with other teachers]. (Maria, *task 6*)

> We also found a professional perspective formula – mixing/integrating the different aspects, routine or no routine, of our

> professional activities: teaching our pupils, training our colleagues and self-development. It was really a success! (Júlia, *task 6*)

Besides investigations, the format of the course – based on a distance education cooperative setting – and its pioneer nature in terms of using ICT and exploring paths of development of a new professional culture seemed also to raise interest in participating in this activity. Regarding this point Júlia and Maria consider their participation as a valuable professional experience.

Reflection. Júlia and Maria enthusiastically discuss papers about curriculum questions and mathematics classroom dynamic, jointly prepare teaching units, reflect about their classes, and discuss general questions that emerge in mathematics teaching and in mathematics teachers' professional culture. They show reflection interests at several levels and make connections among them.

These teachers recognize that this distance teacher education setting led them to write much more than they were used to. Júlia comments that as positive:

> There is one aspect that may be quite particular but that I have felt for a long time [...] It is that we often have very interesting experiences and we discuss issues, and so on, and we always postpone writing, passing them onto paper, recording (…) The fact that this is a distance course [...] compels us into writing... (*interview*)

For them, writing does not appear to be a natural act, perhaps more because of the diversity of demands that they feel at every moment than because of lack of fluency in writing. However, they recognize the importance of writing for their professional practice.

These teachers, in the beginning, did not like the fact that there was a limit in the number of words of the papers that they had to send. With time they began realizing the advantages of this limit, which required a stronger reflection about what they write:

> This can be an aspect that imposes some discipline on us, in the sense that it compels us to reflect, to purify and focus on what is essential in things and even to focus the objectives that would otherwise remain somewhat diffused (Júlia, *interview*)

For them, not all forms of writing assume the same nature. One thing is to write to the teacher educator, another is to write to a discussion forum such the one in this

course. Júlia considers that this element of the virtual setting did not appear to be of natural use, although she recognizes that it may help to create a different professional culture:

> And, on the one hand, the computer may be [as] in the case of the forum, but on the other hand it was an instrument that compelled us towards something that I feel is deeply lacking in Portugal, the existence of written professional culture, sharing, and so on. (*interview*)

Besides writing, the joint activity that these teacher carry out together, the discussions that they held among themselves, and the fact that they cut themselves from the everyday demands, all contributed, in their perspective, to sharing reflections, and constituted a very positive and rewarding professional experience.

Collaboration. Júlia and Maria, with two other accompanying teachers have developed an intense and continued joint work in teacher education for secondary schools. Before the beginning of this course they had established a common weekly working day and they constantly exchanged e-mail messages – according to Maria, an average of ten a day. The four teachers participated in this course but they constituted two groups of two to carry out the tasks. All the remaining activities (discussion of papers, discussion of issues raised in the forum) were developed by the four teachers providing, in their view, a richer and deeper work and strength to keep going:

> I think that in our case, as we are 4=2+2 (...) was really positive because of the discussion, hypotheses and perspectives that were developed or rejected. (Júlia, *tarefa 6*)

> Sometimes it seems that we arrive at Tuesday and it seems that we had not done anything yet, but looking at the papers, seeing what has to be done and to discuss together gives us a lot of strength. If that did not happen we would not have coped. (Júlia, *interview*)

They had developed a previous dynamic of collaborative work and they were quite used to working at a distance, by e-mail; their previous experience enabled them to produce documents almost without seeing each other. The ideas were discussed in the weekly meeting and then, during the rest of the week, by e-mail, they constructed their papers. When one or the other had a personal problem the work was carried by those who had more time, considering the collaborative style of the group: “So, we split ourselves somehow. If one is in bad shape, the other carries the load. There is no discus-

sion, that is, there is no point in arguing. That is very good. It gives me great satisfaction" (Júlia, *interview*)

According to the teachers, their collaborative working dynamic become deeper with this course, which made a positive contribution towards their professional development:

> Working as a group helps to share the competencies of each person and helps to overcome personal inhibitions because of the friendly and responsible commitment of each one with herself and her partner (…) As a pair and individually, we assumed our professional personality, and that is deeply rewarding. (Júlia, *task 6*)
>
> This teamwork attained a greater depth through the study circle, since it allowed us to find, together, a moment to think our own professional development, to work for us. The discussions, sharing reflections, sharing experiences, were very stimulating and enriching (Maria, *task 6*)

They thought a lot together, but they also did a lot of independent work: "Although there is much joint work, there is also a lot of independence and great respect for each other's work [...] There is much putting in common and then doing what is to be done" (Júlia, *interview*).

Discussion

In terms of their professional development, the teachers who participated in this teacher education setting were at quite different starting points and lived rather different experiences. Isaura and Anabela, one experienced and the other a young teacher, had some difficulty in adjusting to the activities proposed. They did not much enjoy reading the papers and carrying out the tasks and the evaluation that they received from the teacher educators was far from positive. They even considered dropping out of the course. A clear mismatch seems to exist between their initial expectations and what they found along the process. They had some curiosity about mathematical investigations but not a high interest and this does not appear to have developed during the virtual course. In task 5 they stress pupils' difficulties in working in open-ended tasks and do not appear to appreciate their potential as a learning environment, emphasising instead the need to structure pupils' work.

In contrast, António and Alda, two young teachers, participated with a high level of interest and sense of responsibility in this distance education course. They were in-

terested in carrying out mathematical investigations in their own classes and wanted to learn about that. The activity that they undertook in this study circle helped them to become aware of different issues related to carrying out investigations with their pupils in the classroom. They had high expectations concerning the discussions that could take place with their colleagues and teacher educators. Although the intensity of the discussions was not exactly what they expected, leading to some frustration, they still went on doing their work carefully until the end of the virtual course.

The third group included two experienced teachers, Júlia and Maria, who had no trouble in adjusting to the spirit and working processes of this teacher education setting. They were quite interested in the topic of mathematical investigations, a kind of task that they used frequently in their classroom practice. They showed a high commitment to act in their classroom as well as in contributing to the development of their professional community. They regarded their participation in the course not so much as an opportunity to learn new ideas but to deepen, clarify, and systematize their perspectives.

The stance regarding reflection and collaboration was also quite different in the three groups of teachers. Isaura and Anabela were willing to question some aspects of their practice, notably those that are more closely related to mathematics topics to be taught but showed little interest in the analysis of issues related to the curriculum or the classroom dynamics, that they regard as essentially "theoretical" and therefore with little relevance for their practice. These two teachers constitute a heterogeneous group if we consider their professional experience and the roles that they had in the past regarding each other. The working dynamic that they developed together seemed to match their expectations and they appeared to have a balanced relationship. These two teachers feel working in pairs is appropriate for preventing distance education to become too isolated and result in an unpleasant experience. That is, they see working together more as an antidote for surviving a demanding and stressful activity rather than a powerful process to deal with an interesting and challenging problem and grow professionally.

Alda and António enjoyed the activity of reading and discussing papers a lot. They showed interest in questioning the most varied aspects of their practice, had curiosity in learning about the contributions that theoretical approaches can provide, and felt compelled to share their experiences with other teachers and teacher educators. These two young teachers, based on their former experience, developed a continuous collaborative work, without setbacks, but also without a clear division of tasks. The ways they value collaborative work led them to assume a high autonomy in their decisions.

Finally, Júlia and Maria revealed wide and varied reflection interests, showing interest in questioning their practice and also in acting at all its levels, from the classroom to intervention in the professional community. Furthermore, to carry out such reflection, they consider the research and professional literature useful, as well as discussions with other teachers and teacher educators and writing about their practice. With solid professional experience and a rich professional curriculum, they developed a strong collaborative work among them, with a clear division of tasks, strongly supported by the use of information technology.

In the face-to-face final session and in the teachers' questionnaires there was a positive evaluation of the virtual teaching education course and also a positive self-evaluation of their participation in this activity. However, we see that the setting proved to be of different value for the different groups of teachers involved. The course was more valuable for those who had a stronger disposition to question themselves and to learn and who were able to better adjust to the processes and spirit of the virtual setting. Figuring out the issues that may arise in conducting mathematical investigations in the classroom, looking at the reactions and processes of pupils and at the role of the teacher, admitting that this may need to be different, according to the circumstances, requires a reflective disposition that was much more visible in some teachers than in others. Another quite distinctive feature in the attitudes of teachers was their stance regarding the research and professional literature, as it addressed useful concepts and issues to think about their own classes. Regarding the nature of their participation, the previous professional knowledge (Elbaz, 1983; Shulman, 1987) of the teachers who became involved in this course seems to be an important differentiating factor. An even stronger element of differentiation was their reflective stance, that is, their disposition to reflect about their actual practice (Oliveira and Serrazina, 2002) and to put into action the consequences of such reflection (Mewborn, 1999). And yet another distinctive feature of the different groups of teachers was the intensity and nature of their working processes, illustrating the variety of meanings concealed in the notion of collaboration (Hargreaves, 1994; Little, 1990).

The virtual teacher education setting involved several tasks and resources that teachers were proposed to do within a certain virtual communication environment. To ease the difficulties of stepping into this environment a face-to-face session was done at the very beginning of the course. Even so, some teachers had some trouble with this environment, either because they missed a certain readiness to become involved in this

kind of task or because they did not feel at ease with virtual interactions. For example, some teachers may wonder what is proper to say in an e-mail message or in a contribution to a discussion forum. Indeed, communication based on the Internet was a problem for Isaura and Anabela, who found it a contrived experience and had trouble using it for professional discussions. In contrast, Julia and Maria interrelated in a very fruitful way both in face-to-face and virtual interactions. This seems to suggest that virtual communities may develop in a very uneven way among teachers, and making sure that everyone is able to establish a productive relationship with the technological media is an important issue in such teacher education settings.

This course is heavily based on reading papers (professional, research) and writing (messages, papers). This is both a strength and a weakness. It is a strength as writing is a powerful way of reflecting, helping teachers to clarify ideas, to look at them from different angles, to come back and revise; the steadiness of the written word also seems to provide more depth to the ideas. It is a weakness since most mathematics teachers are not very used to writing and often regard this as an unnatural activity. For example, for Isaura and Anabela, writing is a quite painful and unfruitful activity; even Julia and Maria recognize that writing may become an important feature of the professional culture of mathematics teachers but they had trouble in finding time to do it; only Alda and António indicated great pleasure in writing and reported no constraint in this activity.

Conclusion

Whereas the final questionnaires pointed to an overall positive evaluation from teachers who participated in this virtual in-service course, the three case studies show strikingly different experiences. Teachers entered with different backgrounds, interests, and expectations and lived different experiences in this activity. We conclude that collaborative distance education can be a valuable learning experience at least for some teachers, supporting their professional development process. Built-in flexibility in the design of these teacher education settings allows for flexible learning (Collis & Moonen, 2001). However, reading, discussing, doing open-ended tasks, reflecting and collaborating are powerful activities but seem to require some sort of readiness. Teacher education programs need to pay attention to the way they negotiate their aims and working processes with participants so that expectations are not frustrated. Also, they need to provide extra forms of helping teachers who want to develop professionally to achieve

that readiness, for example in the form of face-to-face tutorial meetings with teacher educators. That is, flexibility and attention to the needs of the learners must also be an important feature of such settings.

Virtual teacher education courses such as this or with different features may become quite common as virtual learning communities develop and involve more and more teachers. It will be interesting to design in-service experiences with other formats, perhaps combining formal and informal ways of interaction, and consider the problems of negotiating roles and activities and norms of communication. Different forms of teacher education settings may be necessary to address the needs of teachers in different stages of professional development and with different professional interests and levels of familiarity with the technological media.

References

Ackerman, M. (1993). Teacher as reflective practitioner. In D. Owens (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 351-358). Reston: NCTM.

APM (1998). *Matemática 2001: Diagnóstico e recomendações para o ensino e aprendizagem da Matemática* [Mathematics 2001: Diagnostic and recommendations for mathematics teaching and learning]. Lisboa: APM.

Collis, B., & Moonen, J. (2001). *Flexible learning in a digital world.* London: Kogan Page.

Elbaz, F. (1983). *Teacher thinking: A study of practical knowledge.* London: Croom Helm.

Dewey, J. (1997). *How we think*. London: Heath. (originally published 1910)

Ellis, N. (1990). Collaborative interaction for improvement of teaching. *Teaching & Teacher Education, 6*(3), 267-277.

Erickson, F. (1989). Research currents: Learning and collaboration in teaching. *Language Arts, 66*(4), 430-441.

Fonseca, H., Brunheira, L., & Ponte, J. (1999). As actividades de investigação, o professor e a aula de Matemática [mathematical investigations, the teacher and the mathematics class]. *In Actas do ProfMat 99* (pp. 91-101). Lisboa: APM.

Hargreaves, A. (1994). *Changing teaching, changing times: Teachers' work and culture in the postmodern age*. London: Cassel.

Jaworski, B. (1993). The professional development of teachers: The potential of critical reflection. *British Journal of Inservice Education*, *19*, 37-42.

Little, J. W. (1990). The persistence of privacy: Autonomy and initiative in teachers' professional relations. *Teachers College Record, 92*(4), 509-36.

Mason, J. (1991). Mathematical problem solving: Open, closed and exploratory in the UK. *ZDM*, *91*(1), 14-19.

Mewborn, D. S. (1999). Reflective thinking among preservice elementary mathematics teachers. *Journal for Research in Mathematics Education, 30*(3), 316-341.

Oliveira, I., & L. Serrazina (2002). A reflexão e o professor como investigador. In GTI

(Ed.), *Reflectir e investigar sobre a prática profissional* (pp. 30-42). Lisboa: APM.

Olson, M. (1997). Collaboration: En epistemological shift. In H. Christiansen, L. Goulet, C. Krentz, & M. Maeers (Eds.), *Recreating relatioships: Collaboration and educational reform* (pp. 13-25). Albany, NJ: State University of New York Press.

Peter-Koop, A., Santos-Wagner, V., Breen, C., & Begg, A. (Eds.). (2003). *Collaboration in teacher education: Examples from the context of mathematics education*. Dordrecht: Kluwer.

Poincaré, H. (1996). A invenção matemática [Mathematical invention]. In P. Abrantes, L. Leal, & J. Ponte (Eds.), *Investigar para aprender Matemática* (pp. 7-14). Lisboa: Projecto MPT e APM.

Ponte, J. P. (2001). Investigating in mathematics and in learning to teach mathematics. In F. L. Lin & T. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 53-72). Dordrecht: Kluwer.

Ponte, J. P. (in press). Investigating mathematics: A challenge for pupils, teachers, and teacher educators. *Proceedings of the Elementary Education Meeting*, Braga, Portugal.

Ponte, J. P., Boavida, A., Graça, M., & Abrantes, P. (1997). *Didáctica da matemática* [Didactis of mathematics]. Lisboa: DES do ME.

Santos, L. (2002) O trabalho em colaboração entre professores de matemática do ensino secundário num contexto de mudança curricular. *Revista de Educação*, *11*(2), 127-143.

Schön, D. A. (1983). *The reflective practioner: How professionals think in action*. Aldershot Hants: Avebury.

Serrazina, M. L. (1998). *Teachers' professional development in a period of radical change in primary mathematics education in Portugal* (Doctoral dissertation, University of London). Lisboa: APM.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-22.

Skovsmose, O. (2000). Cenários para investigação [Landscapes of investigation]. *Bolema*, *14*, 66-91.

Smylie, M. (1995). Teacher learning in the workplace: Implications for school reform. In T. Guskey & M. Huberman (Eds.), *Professional development in education: New paradigms and practices* (pp. 92-113). New York: Teachers College Press.

Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher, 26*(7), 13-22.

Wood, T. (1994). Patterns of interaction and the culture of the mathematics classroom. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 149-168). Dordrecht: Kluwer.

Zeichner, K. (1993). *A formação reflexiva de professores: Ideias e práticas* [Reflective teacher education: Ideas and practices]. Lisboa: Educa.